Efficient Reiterative Censoring of Robust STAP Using the FRACTA Algorithm[1]

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Abstract — **This paper presents further developments of the FRACTA algorithm [1,2] which has been shown to be robust to nonhomogeneous environments containing outliers. The focus here is on the efficient implementation of the FRACTA algorithm. The key development is a censoring stopping mechanism whereby the number of reiterative steps can be minimized and computation is reduced. We introduce a data-dependent stopping rule that demonstrates excellent results as evidenced by the detection of targets in the KASSPER challenge data cube. We also present some other enhancements to the FRACTA algorithm that further improve both efficiency and performance.**

I. Introduction

In this paper, we develop enhancements to the FRACTA algorithm [1,2], which has been shown to provide robust performance against the presence of outliers in the training data. The enhanced FRACTA algorithm is then applied to the Knowledge-Aided Sensor Signal Processing & Expert Reasoning (KASSPER) challenge data cube [3] with no *a priori* knowledge employed regarding the number, locations, or Dopplers of targets, or the form of the clutter.

Fundamental to most adaptive matched filter (AMF) methods is the accurate estimation of the unknown input covariance matrix. The true covariance matrix is used to find the optimal linear weighting of MN input elements such that the output signal-to-interference ratio is maximized, where N is the number of antenna elements and M is the number of pulses. Due to the lack of knowledge of an external environment, adaptive techniques require a certain amount of data to estimate the $MN \times MN$ input covariance matrix effectively. The amount of data (the number of statistically independent and identically distributed (i.i.d.) samples per input sensor) required so that the performance of the adaptive processor is close (nominally within 3 dB) to the optimum is called the convergence measure of effectiveness (MOE) of the processor. Minimizing the convergence MOE is important since the characteristics of the external interference change rapidly with time in many environments.

Typically for adaptive radar applications, the sample covariance matrix is estimated using training data (called secondary sample data) from range cells close to the primary range cell-under-test (CUT). However, the presence of outliers in the secondary data can skew the covariance matrix estimate such that a true target in the primary range cell is suppressed. Therefore, it is important that all relevant outliers be excised.

A variety of conditions exists wherein outlier data can be present. For example, for the radar problem, sidelobe-clutter discretes could be present in only a few range cells. The temporal covariance matrix of the sidelobe-clutter discretes is much different than say that of the surrounding sea clutter. This problem is closely related to the existence of land-sea clutter interfaces, which cause significant degradation in airborne radar adaptive processing [4]. Other sources of outlier data are the desired targets themselves. For example, if one is trying to detect an individual target adaptively in the presence of a formation of targets (such as an airborne formation), the other desired target returns, located in distinct range cells about the individual desired target with essentially the same velocity vector, can be present in the training data. All of the desired targets have approximately the same desired steering vector. The presence of the desired target returns in the training data can severely degrade the adaptive match filter's performance [5], because the training data is used to estimate a weighting vector which is in the null space of the signal and interference sources that are in the training data. Hence, if a signal that has the desired signal's steering vector is in the training data, the adaptive weight vector may null the desired signal.

In [6], Gerlach developed a robust AMF known as Reiterative Censored Fast Maximum Likelihood (RCFML), whereby outlier data vectors in the training data were censored from the covariance matrix estimate using the Maximum Likelihood Estimation (MLE) [7] setting, along with a version of the Generalized Inner Product (GIP)

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14. ABSTRACT

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measure. The RCFML's convergence performance was shown to be relatively unaffected by the presence of outliers where the interference scenario consisted of homogeneous Gaussian noises plus the outliers. More recently, the Adaptive Power Residue (APR) method has been shown to outperform the GIP method as a discriminant for censoring sample snapshots in the RCFML algorithm [1,2]. The RCFML algorithm using the APR censoring metric is henceforth denoted as the FRACTA algorithm which also employs the Adaptive Coherence Estimate (ACE) along with a local CFAR to screen potential targets.

In this paper we extend the methodology of [1,2] by developing a stopping criterion for censoring samples that is adaptive to the data. This provides a significant savings in computational complexity by eliminating unneeded censoring iterations. Furthermore, since the FRACTA algorithm employs censoring as a first level of detection, the result of fewer censored cells translates into a reduced likelihood for false alarms.

The data base employed to evaluate the performance of the FRACTA algorithm is the KASSPER challenge data cube [3] that was released in April 2002. For the KASSPER data set, the incident power relative to the noise floor on a single antenna element as a function of range and Doppler is illustrated in Fig. 1.

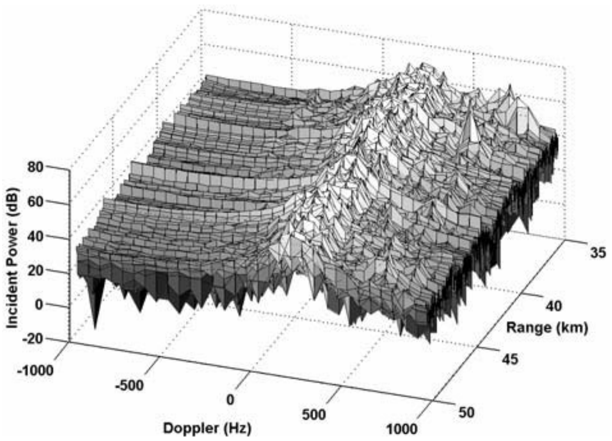

Fig. 1. Incident power for KASSPER data cube.

II. The FRACTA Algorithm

FRACTA is an acronym for the combination: Fast Maximum Likelihood (FML), Reiterative Censoring, the APR censoring metric, Concurrent Block Processing (CBP), Two-weight computation, and the ACE metric. Initial studies indicate that the FRACTA algorithm provides robust detection of targets in nonhomogeneous interference. Reiterative censoring of the FML covariance matrix estimate using APR as the censoring metric (*i.e.* RCFML/APR from [1,2]) has exhibited great potential for accurately culling outliers from data. Essentially, this is accomplished by reiteratively removing from the block of training data the cell that possesses the largest APR, which is defined as the sequence

$$\text{APR:}\quad \left\{\left|\mathbf{s}'\widetilde{\mathbf{R}}^{-1}\mathbf{z}_k\right|^2\right\}_{k=1}^{K} \tag{1}$$

in which $\mathbf{s}$ is the length-MN steering vector, $\mathbf{z}_k$ is the k^{th} length-MN data vector, and $\widetilde{\mathbf{R}}$ is the covariance matrix estimated by FML from the set of K data vectors. Since each reiterative step requires the re-estimation and inversion of the covariance matrix, there is obviously substantial computational benefit in halting censoring when there are no more outliers to be removed from the data.

While some of the pieces of the FRACTA algorithm have been well-studied in the past, Concurrent Block Processing and Two-Weight computation are relatively new concepts. In essence, CBP eliminates the need for guard cells and processes the block of primary data and the surrounding secondary data as a single block of training data which is then reiteratively processed to excise any range cells that are likely to be targets (*i.e.* have a relatively large APR). This results in a set of censored cells (potential targets if in the primary data) and a set of uncensored cells (adaptively determined secondary data). The uncensored cells are used to compute an adaptive weight that is then applied to the censored cells in the primary data block. The total data block (both censored and uncensored cells) is also used to compute an adaptive weight for the uncensored cells. The use of these two weights results in targets standing out dramatically from the suppressed noise and interference.

The detection mechanism used for the FRACTA algorithm consists of three parts: censoring, cell-averaging CFAR (CA-CFAR), and ACE. The censoring step is self-evident. The CA-CFAR is performed on each censored cell in which the value of the average background noise and interference is computed using local uncensored cells. Finally, the ACE is used to determine which potential targets coherently match the steering vector of interest. In this way, the ACE eliminates false targets that may come through the space-time filter sidelobes.

III. Reiterative Censoring Stopping Criterion

The process of reiterative censoring for the FRACTA algorithm is governed by the APR metric. The APR can be decomposed as

$$\left|\mathbf{s}'\widetilde{\mathbf{R}}^{-1}\mathbf{z}_k\right|^2 = \left|\alpha_k\,\mathbf{s}'\widetilde{\mathbf{R}}^{-1}\mathbf{s} + \mathbf{s}'\widetilde{\mathbf{R}}^{-1}\mathbf{n}_k\right|^2 \tag{2}$$

in which we have defined $\mathbf{z}_k = \alpha_k\,\mathbf{s} + \mathbf{n}_k$, where $\mathbf{n}_k$ is a

noise-plus-interference vector and α_k is a complex scalar having non-zero value when a target is present. By assuming that $\mathbf{n}_k$ and $\mathbf{s}$ are not closely matched, which is highly likely in general, and that there are a relatively small number of cells within the block of data having α_k of sufficient magnitude to be detectable, then for those same cells it is very likely that the first term in (2) will dominate as long as $\|\mathbf{n}_k\|$ has roughly the same order of magnitude for all $k = 1,2,\ldots,K$. As the length-*MN* of the data vectors increases, the steering vector is more capable of coherently pulling targets from noise and interference, thus $\|\mathbf{n}_k\|$ can vary more while efficient censoring is still maintained. The need to have a set of $\|\mathbf{n}_k\|$ values of somewhat similar magnitude is so that the noise and interference do not severely skew the ranking of the APR values resulting in an increase in the number of non-targets censored. Furthermore, one can see that as *K* increases, the robustness of the APR ranking will improve due to the fact that the eigenvalue(s) of $\tilde{\mathbf{R}}^{-1}$ corresponding to the shrinking fraction of data vectors similar to $\mathbf{s}$ will be larger, thus the first term of (2) will dominate. This is especially important when several targets are highly concentrated within a given range swath (*i.e.* a column of ground traffic) such that the eigenvalue(s) of $\tilde{\mathbf{R}}^{-1}$ corresponding to $\mathbf{s}$ could be small. In practice, only the $\mathbf{z}_k$ vectors are available so the best that can be done is to ensure that the $\|\mathbf{z}_k\|$ values are within some manageable dynamic range.

With respect to the above discussion, a robust stopping rule has been developed that is adaptive to the data. This is accomplished through the application of a probe data vector that is appended to the data block. The probe vector takes on the form

$$\text{probe:}\quad \alpha_p \mathbf{s} \tag{3}$$

where α_p is a pre-determined magnitude that is set such that the probe vector is nominally detectable (10 – 15 dB above the noise floor). The APR of the probe is found to be $\left|\alpha_p \mathbf{s}'\tilde{\mathbf{R}}^{-1}\mathbf{s}\right|^2$, and whenever this value exceeds the APR for all the true data vectors, censoring is halted. It is a topic of future investigation to approximate accurately the offset in the APR caused by the noise and interference in the second term of (2).

IV. Implementation Alternatives

Besides the probe stopping rule, there are other alternative approaches that can be taken in order to enhance performance and/or reduce the computational load of the FRACTA algorithm. One approach was briefly mentioned earlier and pertains to the way in which data is assigned to blocks for processing. Conventional wisdom leads one to set small block sizes when computing the adaptive weight vector due to the possible non-stationarity of the data. However, when performing censoring using the APR metric, one is searching for data vectors that possess some degree of similarity with a specific steering vector $\mathbf{s}$. Furthermore, at each reiterative step all the remaining uncensored data vectors are contained in the current estimate of the covariance matrix, thus we wish to have the target-like cells to be "nulled the least". In that sense, the non-stationarity of the noise and interference is of secondary concern (as long as the respective $\|\mathbf{n}_k\|$ values are of roughly the same order of magnitude). Therefore, it makes sense to use as many data vectors as possible for censoring in order to drive down the eigenvalues of $\tilde{\mathbf{R}}$ corresponding to target cells thereby greatly increasing the likelihood that the first term of (2) dominates.

Another alternative approach is to replace the FML for covariance matrix estimation with a Loaded Sample Matrix Inverse (LSMI). It is well-known that these two approaches produce very similar results [7]. The important difference is that LSMI can reiteratively re-estimate the covariance matrix after a snapshot has been censored without the need to compute a full matrix inverse. This is accomplished by employing the matrix inversion lemma [8] in which the updated covariance matrix is computed as

$$\tilde{\mathbf{R}}_{m+1}^{-1} = \left(\frac{K-1}{K}\right)\left(\tilde{\mathbf{R}}_m^{-1} + \frac{\left(\tilde{\mathbf{R}}_m^{-1}\mathbf{z}_k\right)\left(\tilde{\mathbf{R}}_m^{-1}\mathbf{z}_k\right)'}{K - \mathbf{z}_k'\tilde{\mathbf{R}}_m^{-1}\mathbf{z}_k}\right), \tag{4}$$

where $\mathbf{z}_k$ is the data vector being censored. This is an approximation to the inverse covariance matrix in which the diagonal loading will change by a factor of $(K-1)/K$ at each reiterative step (and *K* will decrease by 1 each time as well).

Finally, the FRACTA algorithm censoring should be done at full resolution (full integration gain *MN*) to ensure as much accuracy as possible in culling outliers from the data. However, adaptation can be performed at a lower resolution with a graceful degradation in performance. In other words, we can use less than *MN* elements of the respective data vectors to compute the output residue. This is useful because fewer data vectors are needed to estimate a smaller covariance matrix properly, thereby reducing the adverse effects of non-stationarities in estimating an adaptive weight vector. Furthermore, smaller covariance matrices substantially reduce the computational burden of computing a matrix inverse. If we segment the respective data vectors properly (p = 2, 4, etc..), then adaptation can be performed on each segment (using the corresponding segment of the steering vector). The p segmented correlation matrices can be recombined as

$$\tilde{\mathbf{R}}^{-1} = \begin{bmatrix} \tilde{\mathbf{R}}_1^{-1} & 0 & \cdots & 0 \\ 0 & \tilde{\mathbf{R}}_2^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{\mathbf{R}}_P^{-1} \end{bmatrix} \quad (5)$$

to generate the recombined correlation matrix which is employed to compute the APR and ACE.

V. Simulation Results

In order to ascertain the performance of the FRACTA algorithm with the proposed modifications, we apply it to the KASSPER challenge data cube [2] in which M = 32 pulses in the CPI and N = 11 antenna elements. The average power incident upon a single antenna element relative to the noise floor is depicted in Fig. 2 in which we see that the dynamic range is roughly 27 dB. Based on this, the censoring block size is set as K = 1000.

We employ a probe to halt censoring that is 10 dB above the noise floor, which has been normalized to unity. Furthermore, since the clutter returns can be several orders of magnitude greater than the noise floor near the clutter ridge, a maximum number of censored cells is instituted for each Doppler bin and is set to 100. For adaptation, the total block size is set to K_A = 120, of which the 50 cells in the center constitute the primary data block. At the boundaries of the 1000 range cells the secondary data is offset so that the total block size remains constant throughout. Also, we use the reiterative LSMI covariance matrix update from (4).

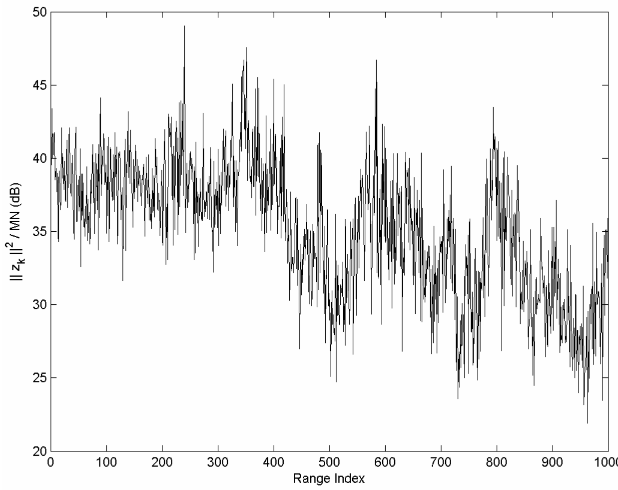

Fig. 2. Range profile of incident power for KASSPER.

The output APR and ACE are illustrated in Fig. 3 and Fig. 4, respectively, in which full resolution (MN) was employed for adaptation. The enhanced FRACTA algorithm is quite able at locating slow-moving targets very close to the peak of the clutter ridge. Of the 32 Doppler bins, only 9 contain true targets and they are all clustered about the clutter ridge. Upon using the probe stopping rule, a total of only 4 cells were censored in all the Doppler bins not containing targets. The result is that when running the FRACTA algorithm on a single processor, the use of the probe stopping along with LSMI is better than 2 times faster than LSMI alone and better than 80 times faster than FML alone.

Figure 5 presents the true targets (black X) along with the cells detected by FRACTA (gray bar). The peak of the clutter ridge is represented by the dashed vertical line near 50 m/s in Doppler. The documentation on KASSPER [2] reports detecting 200 targets with a $P_{FA} = 10^{-6}$ and *a priori* knowledge of the covariance matrix for each range cell. After compactification in range (±2 range cells) and Doppler (±1 Doppler bin) there are 199 detected targets out of the 268 potential targets with a single false alarm. Thus P_F = 1/32,000 = 3.125×10^{-5} is the estimated false alarm probability where there are 32 × 1000 range/Doppler cells. Therefore, for this particular data set, the performance of the FRACTA algorithm is nearly optimal.

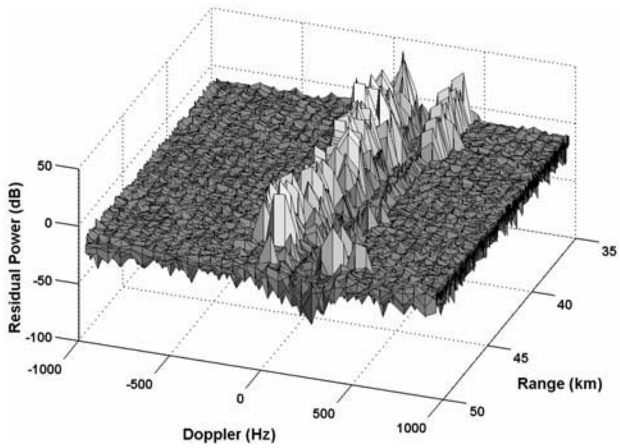

Fig. 3. Output power residue of KASSPER using FRACTA.

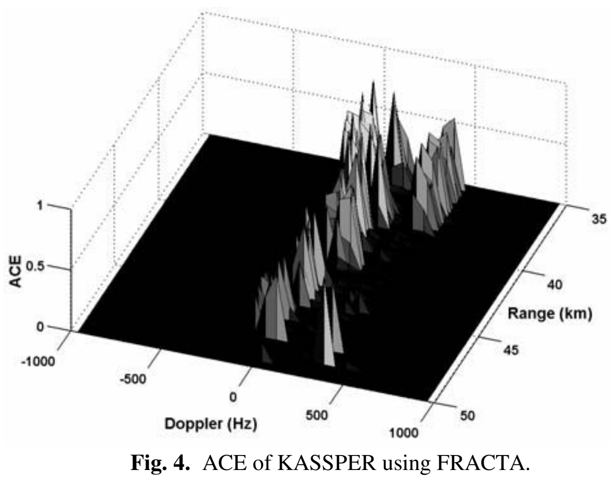

Fig. 4. ACE of KASSPER using FRACTA.

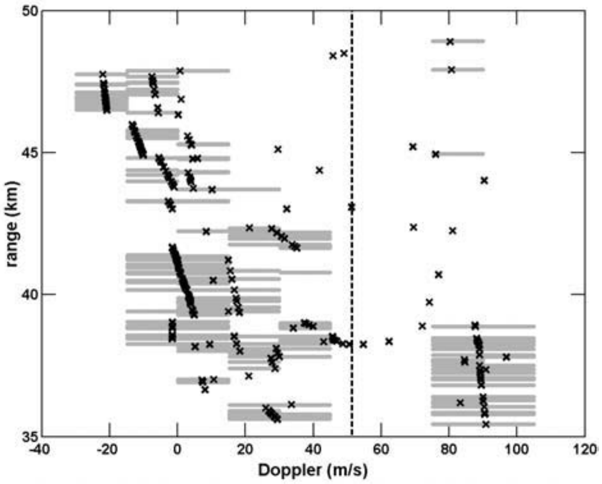

Fig. 5. Detection map of KASSPER using FRACTA.
x = true target, gray = FRACTA detection

If we examine the performance of the FRACTA algorithm when the data vectors have been segmented for the purposes of adaptation, the detection performance degrades gracefully until a breakdown point is reached. As can be seen in Table 1, there is some small loss when going from full resolution to segmentation/recombination by half or by a quarter. When segmenting by an eighth the number of detections drops off more substantially. However, as one increases the number of segments, the size of the covariance matrices decreases and the algorithm becomes more parallelizable and thus computational complexity and speed greatly improve. For practical implementation, this may be a necessary trade-off.

Table 1. Number of detected targets for number of segments

	1 seg.	2 seg.	4 seg.	8 seg.
# targets detected	199	173	167	118

VI. CONCLUSIONS

Effective enhancements for the FRACTA algorithm were presented that enable efficient implementation. A stopping rule for censoring was outlined that can greatly reduce the number of reiterative steps needed to censor all the potential targets in each Doppler bin. The stopping rule is based upon the detection of a nominally detectable target-like probe appended to the data and is well-suited to the ranking structure used to censor cells. Other approaches include a data blocking scheme for APR censoring, a reiterative update approximation to the covariance matrix that substantially reduces the need for matrix inverses, and data vector segmentation for adaptation. Results from application of FRACTA with the above enhancements to the KASSPER challenge data cube indicate a high degree of robustness to dense target environments close to the clutter ridge: target detection capability is nearly the same as when the true covariance matrices are known. A more detailed discussion of the FRACTA algorithm and all its enhancements is given in [9].

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